MATH 2B/5B Prep: Derivatives

1. If $f(x) = 3x^3 + 2\tan(x)$ find $f'(x)$.

Solution:

$$f'(x) = 3\frac{\mathrm{d}}{\mathrm{d}x}x^3 + 2\frac{\mathrm{d}}{\mathrm{d}x}\tan(x) = 3 \cdot 3x^2 + 2\sec^2(x) = 9x^2 + 3\sec^2(x)$$

2. Find the derivatives of $1/x$ and $1/x^2$.

Solution: Writing $1/x$ as x^{-1} and using power rule gives

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x} = \frac{\mathrm{d}}{\mathrm{d}x}x^{-1} = -x^{-2} = \frac{-1}{x^2}$$

Similarly we can write $1/x^2$ as x^{-2} so power rule gives

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x^2} = \frac{\mathrm{d}}{\mathrm{d}x}x^{-2} = -2x^{-3} = \frac{-2}{x^3}$$

3. Compute $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{2x^3 + 3x^2 - x}{x^2}$

Solution: The key here is to start by dividing by x^2:

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{2x^3 + 3x^2 - x}{x^2} = \frac{\mathrm{d}}{\mathrm{d}x}2x + 3 - x^{-1} = 2 + 0 - (-1)x^{-2} = 2 + \frac{1}{x^2}$$